Logic & Implication Relations

Elizabeth Coppock

Handout for Wednesday, November 24th

1 Logic

Implicatures and presuppositions are defined in contrast to entailment. To understand them, it is necessary to understand entailment.

Chierchia and McConnell-Ginet (1990) give the following four definitions for entailment:

- Whenever A is true, B is true
- The information that B conveys is contained in the information that A conveys
- A situation describable by A must also be a situation describable by B
- *A and not B* is contradictory (can't be true in any situation)

To understand why these all mean basically the same thing, we have to understand a little bit of logic. Logic also helps with understanding many other things in pragmatics as well as other areas.

Key concepts: Disjunction, conjunction, negation, equivalence, implication, semantic entailment, tautology, contradiction.

Disjunction. Imagine you are playing a slot machine, and you win if you get three symbols of the same color, or three symbols of the same shape (e.g. a red star, a red square, and a red triangle, or 2 red stars and a blue star). Of course, you also win if you get three symbols of the same color and the same shape. Let:

C = You get three of the same color.
S = You get three of the same shape.

Then we can represent "you get three of the same color or three of the same shape" as:

$$C \vee S$$

The $\vee$ symbol represents "or", a.k.a. *disjunction.* Under what circumstances is $C \vee S$ true?

- When C is true but S is false (e.g. blue diamond, blue diamond, blue triangle)
- When C is false but S is true (e.g. blue diamond, blue diamond, green diamond)
- When C and S are both true (e.g. 3 blue diamonds)

And no other circumstances (e.g. blue diamond, blue triangle, green diamond).

We can represent the meaning of $\vee$ using a *truth table:*

C	S	$C \vee S$
True	True	True
True	False	True
False	True	True
False	False	False

What we have represented here is called *inclusive* disjunction, because it includes the case where *both* of the so-called *disjuncts* are true.

Exclusive disjunction, where only one of the disjuncts can (and must) be true, is defined like this:

C	S	$C \oplus S$
True	True	**False**
True	False	True
False	True	True
False	False	False

This is closer to the intuitive meaning of *or* in “You can have chicken or beef” (but the inference to ‘not both’ is generally thought to be an implicature, while the conventional, semantic meaning of *or* is generally thought to be $\vee$).

Conjunction. Now suppose you had to get all the same shape *and* all the same color in order to win. Then you would need for C *and* S to be true. This is represented:

$$C \wedge S$$

Truth table:

C	S	$C \wedge S$
True	True	True
True	False	False
False	True	False
False	False	False

Negation. The truth table for negation, written $\neg$, is simple. If C is true, then $\neg C$ is false. If C is false, then $\neg C$ is true:

C	$\neg C$
True	False
False	True

What is the negation of $C \wedge S$? To find out, we apply negation to $C \wedge S$:

C	S	$C \wedge S$	$\neg[C \wedge S]$
True	True	True	False
True	False	False	True
False	True	False	True
False	False	False	True

Note: We have to use the brackets [ ] to show that we are applying negation to the conjunction of C and S, rather than C.

Tautology: A logical expression that is true, no matter what values the so-called *propositional variables* it contains take on (propositional variables: e.g. C, S).

You can tell an expression is a tautology by looking at the pattern of Trues and Falses in the column underneath it in a truth table: If they're all true, then it is a tautology.

Here is a tautology: $P \vee \neg P$ (e.g. It is raining or it is not raining):

P	$\neg P$	$P \vee \neg P$
True	False	True
False	True	True

Equivalence. Two propositions are *equivalent* if they are true and false under exactly the same circumstances. Whenever P and Q have the same value (True or False), $P \leftrightarrow Q$ is true:

P	Q	$P \leftrightarrow Q$
True	True	True
True	False	False
False	True	False
False	False	True

P can be said to be equivalent to $\neg\neg P$ because $P \leftrightarrow \neg\neg P$ is a tautology:

P	$\neg P$	$\neg\neg P$	$P \leftrightarrow \neg\neg P$
True	False	True	True
False	True	False	True

$P \leftrightarrow \neg\neg P$ is a tautology because it is always true (as the last column shows).

$[C \vee S] \wedge \neg[C \wedge S]$ is equivalent to $C \oplus S$. You can tell by looking at their pattern of Trues and Falses (both go False, True, True, False):

C	S	$C \wedge S$	$\neg[C \wedge S]$	$C \vee S$	$[C \vee S] \wedge \neg[C \wedge S]$
True	True	True	False	True	False
True	False	False	True	True	True
False	True	False	True	True	True
False	False	False	True	False	False

You can also prove it this way:

C	S	$[C \vee S] \wedge \neg[C \wedge S]$	$C \oplus S$	$[[C \vee S] \wedge \neg[C \wedge S]] \leftrightarrow [C \oplus S]$
True	True	False	False	True
True	False	True	True	True
False	True	True	True	True
False	False	False	False	True

Implication. Returning to our game, let W = you win. You win if and only if you get all the same color or all the same shape. But it is also true that you win if you get all the same color. We can represent "If you get all the same color, then you win" as follows:

$$C \rightarrow W$$

Suppose there have been some questions about whether the slot machine is functioning correctly, and we want to check whether this claim is true. There are several cases to check:

- You get all the same color and you win. ✓
- You get all the same color and you don't win. **No!**
- You don't get all the same color but you still win (presumably you got all the same shape). ✓
- You don't get all the same color and you don't win (presumably you didn't get all the same shape). ✓

Thus the truth table for $C \rightarrow W$ is:

C	W	$C \rightarrow W$
True	True	True
True	False	False
False	True	True
False	False	True

Fact: $P \rightarrow Q$ is equivalent to $\neg P \vee Q$:

P	Q	$\neg P$	$\neg P \vee Q$
True	True	False	True
True	False	False	False
False	True	True	True
False	False	True	True

Semantic entailment. P semantically entails Q if and only if: whenever P is true, Q is true. Notation:

$$P \vDash Q$$

The same symbol can be used like this, meaning "P is a tautology":

$$\vDash P$$

In propositional logic, P semantically entails Q if and only if $P \rightarrow Q$ is a tautology:

$$P \vDash Q \text{ if and only if } \vDash P \to Q$$

Contradiction. Two logical expressions are contradictory if they are never true at the same time. Example: P and $\neg P$.

P	$\neg P$	$P \wedge \neg P$
True	False	False
False	True	False

Proof that they are contradictory: Their conjunction is always false.

A contradictory pair: $P \to Q$, and $P \wedge \neg Q$.

P	Q	$\neg Q$	$P \wedge \neg Q$
True	True	False	False
True	False	True	True
False	True	False	False
False	False	True	False

This is the opposite pattern of $P \to Q$:

P	Q	$P \to Q$
True	True	True
True	False	False
False	True	True
False	False	True

So their conjunction will always be false:

P	Q	$P \to Q$	$P \wedge \neg Q$	$[P \to Q] \wedge [P \wedge \neg Q]$
True	True	True	False	False
True	False	False	True	False
False	True	True	False	False
False	False	True	False	False

When the conjunction of two logical expressions is always false, the two expressions are contradictory.

On the other hand, $P \to Q$ is equivalent to: $\neg[P \wedge \neg Q]$.

So $P \to Q$ means, it can never be the case that P is true while Q is false.

Another way of putting it: $P \to Q$ is a tautology if and only if P and $\neg Q$ are contradictory.

Back to entailment. Recall Chierchia and McConnell-Ginet's (1990) four definitions for 'A entails B':

- Whenever A is true, B is true

- The information that B conveys is contained in the information that A conveys
- A situation describable by A must also be a situation describable by B
- *A and not B* is contradictory (can't be true in any situation)

These can all be understood as:

$$A \vDash B$$

or

$$\vDash A \rightarrow B$$

which is equivalent to:

$$A \text{ and } \neg B \text{ are contradictory}$$

2 Implication Relations

2.1 Implicatures vs. Entailments

Test 1: Defeasibility/Cancellation. It follows from the view of entailment given above that entailments are *not defeasible*: If A entails B, and A is true, then B must be true. To say A and then $\neg B$ would be contradictory.

Implicatures are different. They do not follow from the *sentence itself,* but rather from the *utterance* of the sentence, and the context in which they are uttered. In other words, *speakers implicate,* but *sentences/propositions entail.*

Since what is implicated does not follow logically, implictures are *defeasible.* In other words, implicatures can be *cancelled.*

Entailment or implicature?

(1) a. Mary used to swim a mile daily.
    b. Mary no longer swims a mile daily.

(2) a. After Hans painted the walls, Pete installed the cabinets.
    b. Hans painted the walls.

Applying the cancellation test:

(3) (1a)... And she still does. $\leftarrow$ Perfectly fine

(4) (2a)... But Hans didn't paint the walls. $\leftarrow$ Huh???

A variant: expressing ignorance about the entailment:

(5) (1a)... I wonder whether she still does. $\leftarrow$ Perfectly fine

(6) (2a)... I wonder if Hans painted the walls. $\leftarrow$ Huh??? Contradiction.

Entailment or implicature? Use the cancellation test.

(7) a. Today is sunny.
    b. Today is warm.

(8) a. Jane ate oatmeal for breakfast this morning.
    b. Jane ate breakfast this morning.

(9) a. Mary is an Italian violinist.
    b. Some Italian is a violinist.

(10) a. Joan likes some of her presents.
     b. Joan doesn't like all of her presents.

(11) a. If you finish your vegetables, I'll give you dessert.
     b. If you don't finish your vegetables, I won't give you dessert.

Test 2: Reinforcement. Implicatures can also be *reinforced*:

(12) Mary used to swim a mile daily, and she still does.

(13) If you finish your vegetables, I'll give you dessert, and if you don't finish your vegetables, I won't give you dessert.

Not entailments:

(14) #Mary had oatmeal for breakfast this morning, and she had breakfast this morning.

(15) #Mary is an Italian violinist, and some Italian is a violinist.

2.2 Presuppositions vs. Entailments

Recall that presuppositions are assumptions that the speaker makes, and assumes that the hearer shares. "If A presupposes B, then A not only implies B but also implies that the truth of B is somehow taken for granted, treated as uncontroversial" (Chierchia and McConnell-Ginet 1990, p. 28).

Like entailments, presuppositions are not defeasible:[1]

(16) a. Sue stopped smoking.
     b. Sue used to smoke.

(17) (16a)... But she never smoked. $\leftarrow$ Contradiction!

We need other tests to distinguish between presupposition and entailment.

[1] Except under negation: *Sue didn't stop smoking – She never smoked!*

Test 1: Projection. Unlike entailments, presuppositions *project*. "If A presupposes B, then to assert A, deny A, wonder whether A, or suppose A – to express *any* of these attitudes toward A is generally to imply B, to suggest that B is true and, moreover, uncontroversially so. That is, considering A from almost any standpoint seems already to assume or presuppose the truth of B; B is part of the background against [which] we (typically) consider A" (C&MG, p. 28).

(18) a. Joan regrets getting her Ph.D. in linguistics.
b. Joan doesn't regret getting her Ph.D. in linguistics.
c. Does Joan regret getting her Ph.D. in linguistics?
d. If Joan regrets getting her Ph.D. [in] linguistics, she should consider going back to graduate school in computer science.

All imply:

(19) Joan got her Ph.D. in linguistics.

Another example:

(20) a. All Mary's lovers are French.
b. It isn't the case that all Mary's lovers are French.
c. Are all Mary's lovers French?
d. If all Mary's lovers are French, she should study the language.

All imply:

(21) Mary has (three or more?) lovers.

Thus presupposition "involves not just a single implication but a family of implications" (C&MG, p. 29).

In other words, presuppositions *project* through negation, etc.

Test 2: *Hey, wait a minute!*

(22) A: Joan regrets getting her Ph.D. in linguistics.
B: Hey, wait a minute. I had no idea that Joan did her Ph.D. in linguistics.

(23) A: Sue stopped smoking.
B: Hey, wait a minute. I had no idea that Sue ever smoked.

(24) A: Mary is an Italian violinist.
B: #Hey, wait a minute. I had no idea that Mary was Italian.

Presupposition or entailment?

(25) a. The mathematician who proved Goldbach's Conjecture is a woman.
b. Somebody proved Goldbach's Conjecture.

(26) a. The flying saucer came again.
b. The flying saucer has come sometime in the past.

(27) a. The flying saucer came yesterday.
b. The flying saucer has come sometime in the past.

(28) a. Mary has written four books.
b. Mary has written at least three books.

(29) a. I want to read all of the books that Mary has written.
b. Mary has written at least three books.

(30) a. It was Henry who kissed Rosie.
b. Someone kissed Rosie.

(31) a. Henry kissed Rosie.
b. Someone kissed Rosie.

(32) a. Obama has also invited Angela.
b. Obama invited someone other than Angela.

(33) a. Obama has invited Angela and Ben.
b. Obama invited someone other than Angela.

(34) a. If the notice had said 'mine-field' in English as well as Welsh, we would never have lost poor Llewellyn.
b. The notice did not say 'mine-field' in English.