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Due: October 19th, 2011
Time: 14:30–16:00

Compositional Semantics
Heinrich Heine University
Winter Semester 2011/12
Room: 25.22-U1.72

Problem Set 1: Logic and Set Theory

Note: Please print out your answers and bring them to class on October 19th so we can discuss them all together!

Also: Please label your answers A1, A2, ... B1, B2,...

A. Read: Kearns, *Semantics,* sections 2.1 and 2.2 (pp. 25-35).

1. What is a *proposition*?
2. What does it mean for something to be *truth-functional*?
3. Why isn't the word "because" truth-functional?
4. What is a *connective*?
5. What is something that is truth-functional, but not a connective?
6. Give the truth tables for conjunction, inclusive disjunction, negation, and material implication.
7. Kearns gives a truth table for inclusive disjunction but not exclusive disjunction. Based on how she describes it, what is the truth table for exclusive disjunction? You can use either the symbol '$\oplus$' or 'XOR' to represent exclusive disjunction.
8. Exercise C, p. 48.
9. A proposition is a *tautology* if it is always true, regardless of what values the variables it contains take on. For example, '$P \vee \neg P$' is a tautology:

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

because there is a 'T' in every row in the column for '$P \vee \neg P$' in the truth table (the last column). Decide whether the following is a tautology:

$$[P \rightarrow Q] \iff [\neg Q \rightarrow \neg P]$$

In other words, is '$P \rightarrow Q$' is equivalent to '$\neg Q \rightarrow \neg P$'?

10. Two propositions are *contradictory* if they can never be true at the same time. For example, 'P' and '$\neg P$' are contradictory. You can tell by looking at the final column in the truth table for their conjunction, '$P \wedge \neg P$', which has 'F' in every row:

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

Are '$P \rightarrow Q$' and '$P \wedge \neg Q$' contradictory? Show using a truth table.

B. Read: Heim and Kratzer (1998), chapter 1.

1. Let $A = \{a, b, c\}$ and $A' = \{A\}$. (i) Is A a subset of A'? (ii) Is $\{A\}$ a subset of A'? (iii) Is A a member of A'? (iv) Is a a member of A'?
2. List all of the subsets of A. Don't forget the empty set!
3. Let $A = \{a, b, c\}$ and $B = \{a, d, e\}$. (i) What is the intersection of A and B? (ii) What is the union of A and B? (iii) What is the complement of A in B?
4. True or false: $\langle a, b\rangle = \langle b, a\rangle$. Why?
5. True or false: $\{a, b\} = \{b, a\}$. Why?
6. Under what circumstances is a relation a function?
7. Which of the following relations is a function:
   (a) $\begin{bmatrix} a & \rightarrow & b \\ c & \rightarrow & b \\ d & \rightarrow & c \end{bmatrix}$ (b) $\begin{bmatrix} a & \rightarrow & a \\ c & \rightarrow & b \\ d & \rightarrow & c \end{bmatrix}$
8. (i) What is the domain of the relation (a)? (ii) What is its range?
9. Give a function f such that $f(0) = 1$, $f(1) = 0$, $f(2) = 1$, $f(3) = 0$, $f(4) = 1$, and $f(5) = 0$.